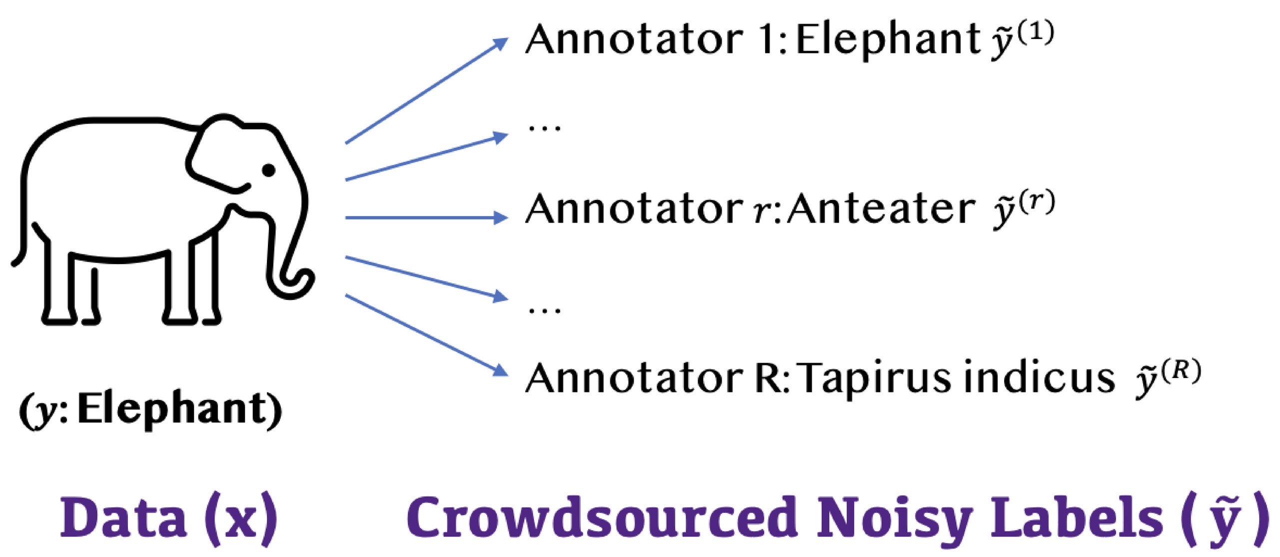
Label Correction of Crowdsourced Noisy Annotations with an Instance-Dependent Noise Transition Model

Hui Guo [1], Boyu Wang [1], Grace Y. Yi [1,2]

[1] Department of Computer Science, University of Western Ontario [2] Department of Statistical and Actuarial Sciences, University of Western Ontario

Introduction

- **Crowdsourcing:** each data item is labeled by **multiple annotators** with **diverse expertise**



- **Noisy training data:** $\mathcal{D} = \{\mathbf{x}_i, \tilde{y}_i^{(1)}, \dots, \tilde{y}_i^{(R)}\}_{i=1}^N$
  - R: number of annotators
  - $\mathbf{x}_i$: input data
  - y_i: unobserved true label
  - $\tilde{y}_i^{(r)}$: the label given by the rth annotator with $r \in \{1, \dots, R\}$
- **Goal:** learn a classifier which correctly labels the new input data

Motivation

- **Assumption:** the R annotators **independently** label the instances
- **Noisy label generation process:**

$$\mathbb{P}(\tilde{y}^{(1)}, \dots, \tilde{y}^{(R)}|\mathbf{x}) = \prod_{r=1}^{R} \mathbb{P}(\tilde{y}^{(r)}|\mathbf{x})$$

$$= \prod_{r=1}^{R} \sum_{k \in \mathcal{Y}} \left\{ \mathbb{P}(\tilde{y}^{(r)}|y = k, \mathbf{x}) \; P(y = k|\mathbf{x}) \right\}$$

base model $h(\cdot;\vartheta)$ (true label predictor)

instance-dependent noise transition matrix for the rth annotator
$f_0^{k,r}(\mathbf{x})$: distribution of $\tilde{y}^{(r)}|\{y = k, \mathbf{x}\}$, modeled by $f_{\boldsymbol{\theta}}^{k,r}(\mathbf{x})$

- **Issues about instance-dependent transition matrices:**
  - Most available methods require the *instance independent* assumption: $\mathbb{P}(\tilde{y}^{(r)}|y = k, \mathbf{x}) = \mathbb{P}(\tilde{y}^{(r)}|y = k)$; however, the instance dependent assumption is **more realistic**
  - Modeling the instance-dependent transition matrix is **challenging** and remains relatively **less explored**
  - **Theoretical characterization** of the distance of the noise model and the true transition matrix remains absent in the literature

Theoretical Analysis and Methodology

Instance – Dependent Noise Transition Matrices

- **Sparse Bayesian network:**
  - Deploy a set of **(δ-pseudo) anchor points** $\overline{\mathcal{D}}_0$ learned from noisy training data
    - An instance $\mathbf{x}$ is defined to be an (δ-pseudo) anchor point of class k if $\mathbb{P}(y = k|\mathbf{x}) = 1$ ($\mathbb{P}(y = k|\mathbf{x}) \geq \delta$)
    - The subsample size n of $\overline{\mathcal{D}}_0$ is relatively small compared to the main sample size N
  - Employ a **hierarchical spike and slab prior** on the network parameters
    - Sparse Bayesian network $f_{\boldsymbol{\theta}}^{k,r}$ with $\boldsymbol{\theta} \in \Theta$
- **Posterior consistency result:**
  - The sparse noise transition model is close to the underlying true transition matrix with respect to the Hellinger distance under mild conditions

Theorem 1: Posterior consistency

Let $d(\cdot,\cdot)$ denote the Hellinger distance. Under regularity conditions, there exists a sequence of constants $\{\epsilon_n^2\}_{n=1}^{\infty}$ satisfying $\lim_{n\to\infty} \epsilon_n = 0$ and $\lim_{n\to\infty} n\epsilon_n^2 = \infty$, such that for any $k \in \{1, \dots, K\}$ and $r \in \{1, \dots, R\}$, with probability tending to 1, the posterior measure satisfies

$$\Pi\left\{\boldsymbol{\theta} \in \Theta : d(f_{\boldsymbol{\theta}}^{(k,r)}, f_0^{(k,r)}) > M_n\epsilon_n|\overline{\mathcal{D}}_0\right\} \to 0 \text{ as any } M_n \to \infty.$$

Pairwise Likelihood Ratio Test for Label Correction

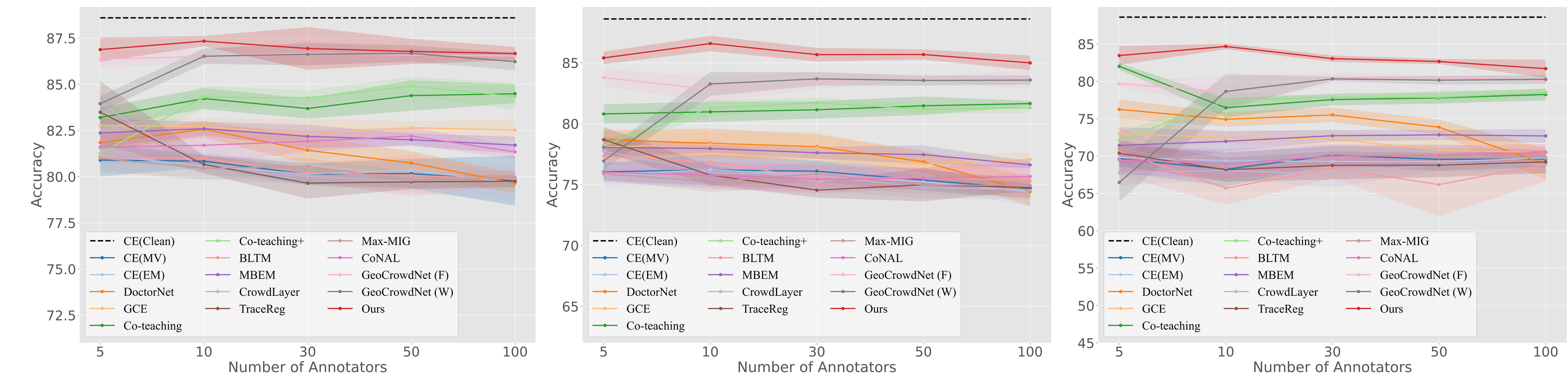
- **Reformulate the label correction process:**
  - Selecting the label for the instance $\mathbf{x}_i$ from $\{g, g'\}$, is **equivalent** to choosing from the two competitors $\mathbb{P}(\tilde{\mathbf{y}}|y = g, \mathbf{x}_i)$ and $\mathbb{P}(\tilde{\mathbf{y}}|y = g', \mathbf{x}_i)$, where $1 \leq g < g' \leq K$
  - **Hypothesis testing:** $H_g : \tilde{\mathbf{y}}_i|\{y_i, \mathbf{x}_i\} \sim \mathbb{P}(\tilde{\mathbf{y}}|y = g, \mathbf{x}_i)$ versus $H_{g'} : \tilde{\mathbf{y}}_i|\{y_i, \mathbf{x}_i\} \sim \mathbb{P}(\tilde{\mathbf{y}}|y = g', \mathbf{x}_i)$
- **Label correction method:**
  - (Neyman-Pearson Lemma) **Set the estimated label of $\mathbf{x}_i$** to be $\overline{y}_i = g$ if

$$\frac{\hbar_{i,g} \prod_{r=1}^{R} \prod_{l=1}^{K} \left\{\tau_{i,gl}^{(r)}\right\}^{\mathbf{1}(\tilde{y}_i^{(r)} = l)}}{\hbar_{i,g'} \prod_{r=1}^{R} \prod_{l=1}^{K} \left\{\tau_{i,g'l}^{(r)}\right\}^{\mathbf{1}(\tilde{y}_i^{(r)} = l)}} > \Omega \text{ for any } g' \neq g$$

  - $\hbar_{ig}$: class prior for the ground truth label for the ith task for $g \in \{1, \dots, K\}$
    $\Longrightarrow$ the predictions of base classifiers
  - $\tau_{i,kl}^{(r)}$: the lth element of $f_{\boldsymbol{\theta}}^{(k,r)}(\mathbf{x}_i)$ for $k, l \in \{1, \dots, K\}$ and $r \in \{1, \dots, R\}$
    $\Longrightarrow$ the maximum a posteriori (MAP) estimate
  - Ω: pre-specified threshold
- **Theorem 2:**
  - Information-theoretic bounds on the Bayes error

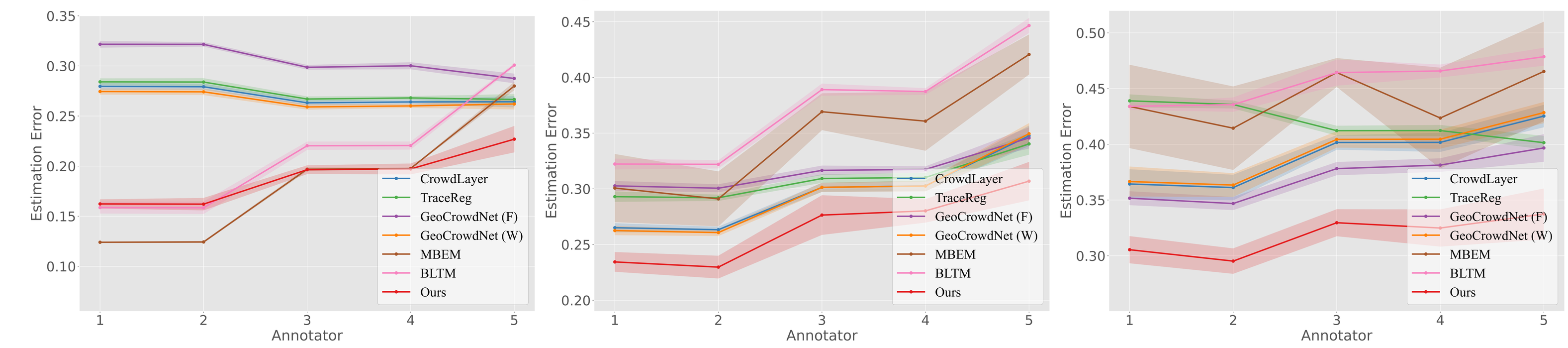
Empirical Results

Empirical Results on CIFAR10 with Varying Number of Annotators:



(a) IDN-LOW (b) IDN-MID (c) IDN-HIGH

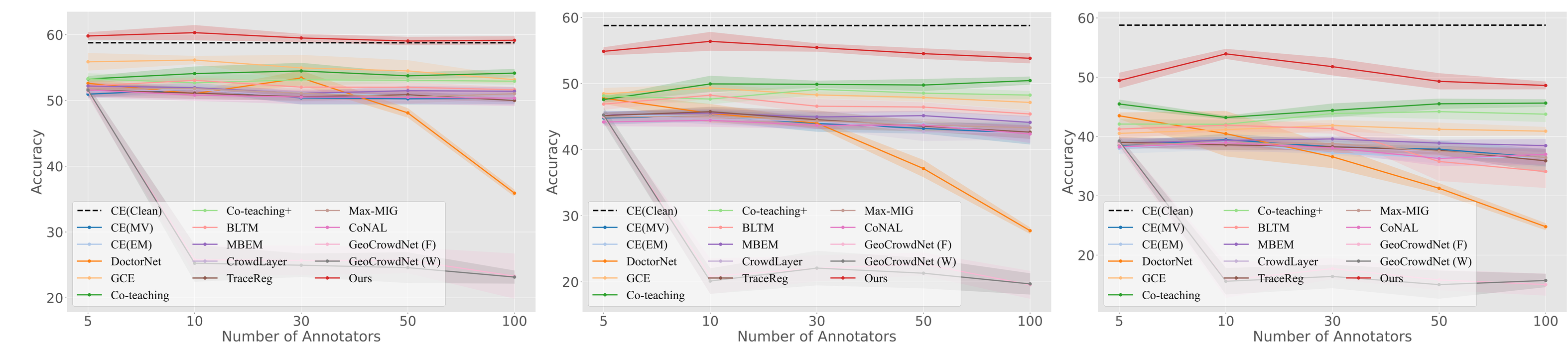
Figure 1: Average accuracy.



(a) IDN-LOW (b) IDN-MID (c) IDN-HIGH

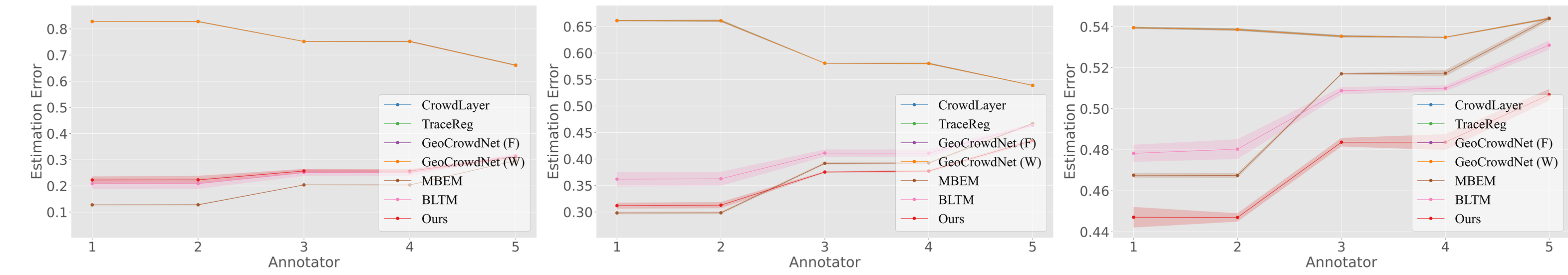
Figure 2: Average estimation error of noise transition matrices.

Empirical Results on CIFAR100 with Varying Number of Annotators:



(a) IDN-LOW (b) IDN-MID (c) IDN-HIGH

Figure 3: Average accuracy.



(a) IDN-LOW (b) IDN-MID (c) IDN-HIGH

Figure 4: Average estimation error of noise transition matrices.

With varying number of annotators, the proposed method
- achieves **the highest average test accuracy**;
- leads to **smaller estimation error** in most of the cases, especially when the noise rate is high.

Contributions

1. We explore the challenging problem of learning with instance-dependent crowdsourced noisy annotations;
2. We formulate the annotator-specific noise transition matrix in the **Bayesian framework**;
3. We **theoretically characterize the closeness** of the proposed sparse Bayesian model and the underlying annotator confusions with respect to the Hellinger distance;
4. We develop a novel **label correction algorithm** by aggregating the noisy annotations using the pairwise likelihood ratio test, and identify **information-theoretic bounds** on the Bayes error;
5. **Numerical experiments** demonstrate that the proposed method outperforms the competing methods.